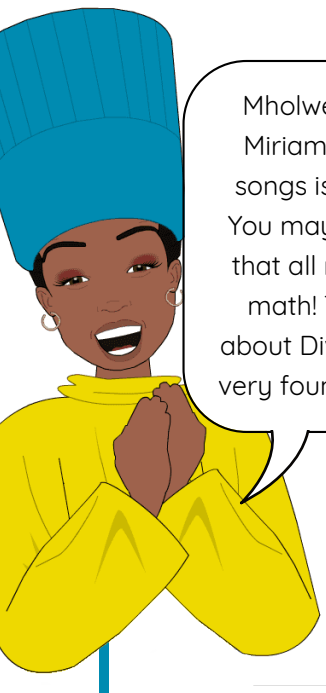
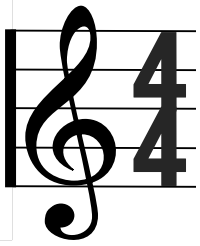


Mholweni! Hello! My name is Miriam Makeba, and writing songs is my favorite pastime. You may be surprised to know that all music theory is simply math! Today we're learning about Divisibility, which is at the very foundation of composition.



Time Signatures: The top number tells us how many beats are in a bar, and the bottom number tells us what length the beat is. The total number of beats in the song must be divisible by the top number!

DIVISIBILITY

GAIM Newsletter Activity for **Vol. 1** | **Issue 1**

In today's activity, we are going to learn how to tell if one number can be divided by another number without having to actually divide them! A whole number is "divisible" when you can divide it by another whole number without any remainder.

Example 1

$\frac{42}{7} = 6$ since this is a whole number, we can say that 42 **"is divisible by"** 7

$\frac{42}{5} = 8\ R2$ since this is not a whole number, 42 **"is not divisible by"** 5

BEFORE WE BEGIN

In order to understand divisibility, we'll first have to go over some concepts that we'll use throughout the activity. We assume that you already know how to multiply and how to divide with remainder, but we'll brush up on the concepts below. Make sure you understand them one at a time, as they build on each other.

- An **integer** is a whole number that can be positive, negative, or 0. For instance, numbers like 47, 3, 0, -8, and -13 are integers, but the numbers $\frac{2}{3}$, 3.14 and 4R1 are not.
- One integer is **divisible** by another when you divide the first by the second and the result is an integer. So, 4 is divisible by 2 because 4/2 = 2 and 21 is divisible by 7 because 21/7 = 3. However, 13 is not divisible by 2, because 13/2 = '6 R1' or '6 and 1/2'.
- An integer greater than 1 is **prime** when it is divisible by exactly two integers: 1 and itself. So, for example, 17 can only be divided by 1 and 17, therefore 17 is prime. The number 16 is divisible by 1, 2, 4, 8, and 16, so 16 is not prime.
- A **composite** number is a positive integer that can be formed by multiplying two smaller positive integers. For example, 15 is composite since **3** x **5** = **15**.
- Any integer greater than 1 is either a prime number, or can be made by multiplying prime numbers together.
- If two numbers are divisible by another number, then their sum is divisible by that number. For example, 30 is divisible by 5 and 45 is divisible by 5, so (30+45) = 75 is divisible by 5.

GOAL:

Develop the divisibility rules for when a given integer is divisible by 2, 4, and 8, so that we don't have to do lots of division to know whether a number is divisible by them!

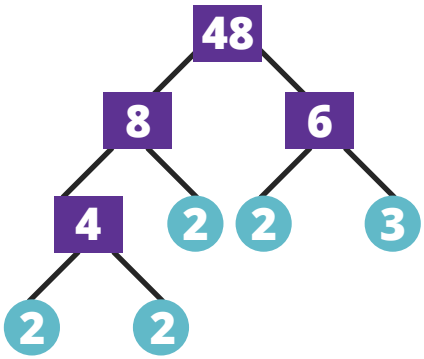
Factor trees look an awful lot like the Baobab Tree...

...if you turn them upside down! Africans call the Baobab the "tree of life"; it is the largest and has the longest lifespan of any other flowering plant!

Prime Factorization using
FACTOR TREES

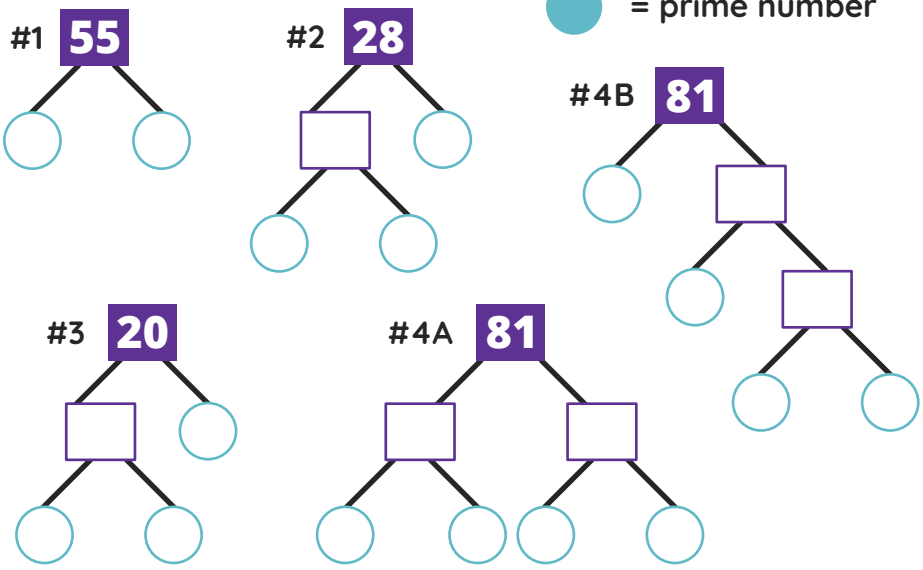
Let's make factor trees! Divisibility is about numbers making other numbers, and so is factorization. Create factor trees by dividing the numbers up into smaller numbers. Each set of smaller numbers that you create should multiply together to create the number above it, like the **Example 2** diagram to the right!

Example 2



TRY IT YOURSELF!

Instructions: Can you create factor trees out of the numbers below? Do you notice any trends? Many are able to be divided more than once, and you might notice a lot of the same prime factors. You can now say that each of the top numbers is "divisible" by the bottom numbers!



What's next?

On the next few pages, we'll learn about some of the rules that make determining the divisibility of even a five digit number as easy as a quick glance!

DIVISIBILITY RULES

2, 4, & 8

RULE FOR #2

A number is divisible by 2 **if the last digit is divisible by 2**. You'll notice that "2" was a popular prime number in the factor trees on the previous page—any of those trees with a "2" in a circle is divisible by 2.

Which of the following numbers are divisible by 2? (Circle your answer.)

38 **465** **1234**

WHY?

Consider the clever method for rewriting a given integer so that we can easily identify **the digit in the one's place**. First, we write every integer above as **A** x 10 + **B,** where **B** is the digit in the one's place.

38	**465**	**1234**
AB	**AB**	**AB**
3 x 10 + **8**	**46** x 10 + **5**	**123** x 10 + **4**

Notice that 10 is divisible by 2, since 10 = 2 x 5. Thus, any multiple of 10 is divisible by 2. So, if **B** is divisible by 2 then **AB** is divisible by 2.

TRY IT YOURSELF!

Only some of the below numbers are divisible by 2. Test out your knowledge using the rule above to figure out which numbers are!

55 **128** **1367** **123098**

RULE FOR #4

A number is divisible by 4 **if the last two digits are divisible by 4.**

Which of the following numbers are divisible by 4? (Circle your answer.)

2644 **52102**

WHY?

Let's make this hands-on! If each number above is **ABC**, write them both as **A** x 100 + **BC**.

2644
ABC

26 x 100 + **44**

52102
ABC

521 x 100 + **02**

100 is divisible by 4, so any multiple of 100 is divisible

$\frac{44}{4} = 11$

$\frac{02}{4} = 0\ R2$

Note: If there are only two digits, those are the digits you test!

Recall, if any two numbers are divisible by 4, then the sum of those numbers is divisible by 4. Since 4 divides 100, then 4 also divide **A** x 100. So, if 4 divides **BC** then 4 divides **ABC**.

TRY IT YOURSELF!

Only some of the below numbers are divisible by 4. Test out your knowledge using the rule above to figure out which numbers are!

3240 **5482** **168** **6784**

RULE FOR #8

A number is divisible by 8 **if the last three digits are divisible by 8**.

Which of the following numbers are divisible by 8? (Circle your answer.)

6152 **11816** **913206**

Need a hint? Try writing each of the above as **A x 1000 + BCD**.

Let's take it further!

Can you write a rule based on the above?

WHY?

Recall, if any two numbers are divisible by 8 then the sum of those two numbers is divisible by 8. Any number **ABCD** can be written as **A x 1000 + BCD** and 1000 is divisible by 8, then we know **ABCD** is divisible by 8 if **BCD** is divisible by 8.

6152	11816	913206
ABCD	ABCD	ABCD
6 x 1000 + 152	11 x 1000 + 816	913 x 1000 + 206

1000 is divisible by 8 because 1000 = 8 x 125, so any multiple of 1000 is divisible by 8

$\frac{152}{8} = 19$ $\frac{816}{8} = 102$ $\frac{206}{8} = 25\ R6$

TRY IT YOURSELF!

The next page is a worksheet covering rules 2, 4, and 8. Test out your knowledge using the rules and tricks you learned in this activity on the next few pages. If you printed the worksheet yourself, the answers are upside down on the back.

Divisibility by 2, 4, & 8

QUICK REVIEW

Place a checkmark in the box if the top number is divisible by the number next to the box. Do you notice a pattern? Why?

1) **21**
- by 2 ☐
- by 4 ☐
- by 8 ☐

2) **178**
- by 2 ☐
- by 4 ☐
- by 8 ☐

3) **1124**
- by 2 ☐
- by 4 ☐
- by 8 ☐

4) **112000**
- by 2 ☐
- by 4 ☐
- by 8 ☐

Complete the following word problems. If the words are confusing you, focus first on the numbers. You may find it easier to check all of the numbers' divisibility first!

Problem #1

The "Two, Four, Eight, All we do is Appreciate Company" produces food items **equally** in either 2, 4, or 8 flavors. What is the maximum number of flavors (2, 4, or 8) the company can choose to produce if:

1A. They produce 2908 jars of pasta sauce every year?

1B. They produce 14,000 boxes of dried pasta every year?

1C. They produce 1998 packages of meatballs every year?

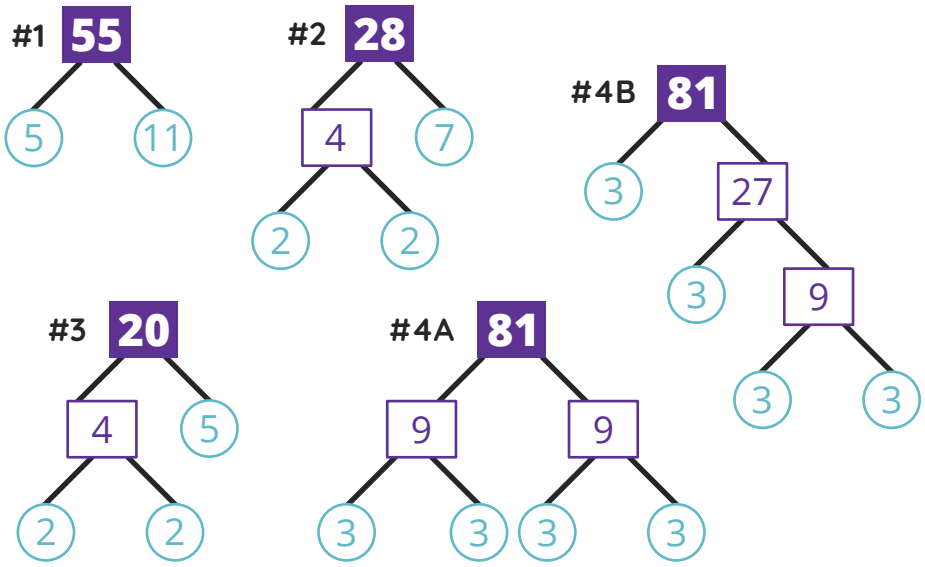
Problem #2

The city of Twofoureightville has 112,748 residents that voted in the city election with either 2, 4, or 8 candidates. After tallying all of the votes, it was discovered that each candidate had exactly the same number of votes! What is the maximum number of candidates possible (2, 4 or 8) in for this to have occurred?

TRY IT YOURSELF!

You can practice divisibility with any number you can dream! Is your age divisible by 2, 4, or 8? How about your parents' phone numbers? What about the street number in your address? The sky is the limit!

FACTOR TREES ANSWERS



QUICK REVIEW ANSWERS

1) **21**

- by 2 ☐
- by 4 ☐
- by 8 ☐

2) **178**

- by 2 ■
- by 4 ☐
- by 8 ☐

3) **1124**

- by 2 ■
- by 4 ■
- by 8 ☐

4) **112000**

- by 2 ■
- by 4 ■
- by 8 ■

WORD PROBLEM ANSWERS

1A. 4 flavors
1B. 8 flavors
1C. 2 flavors
2. 4 candidates