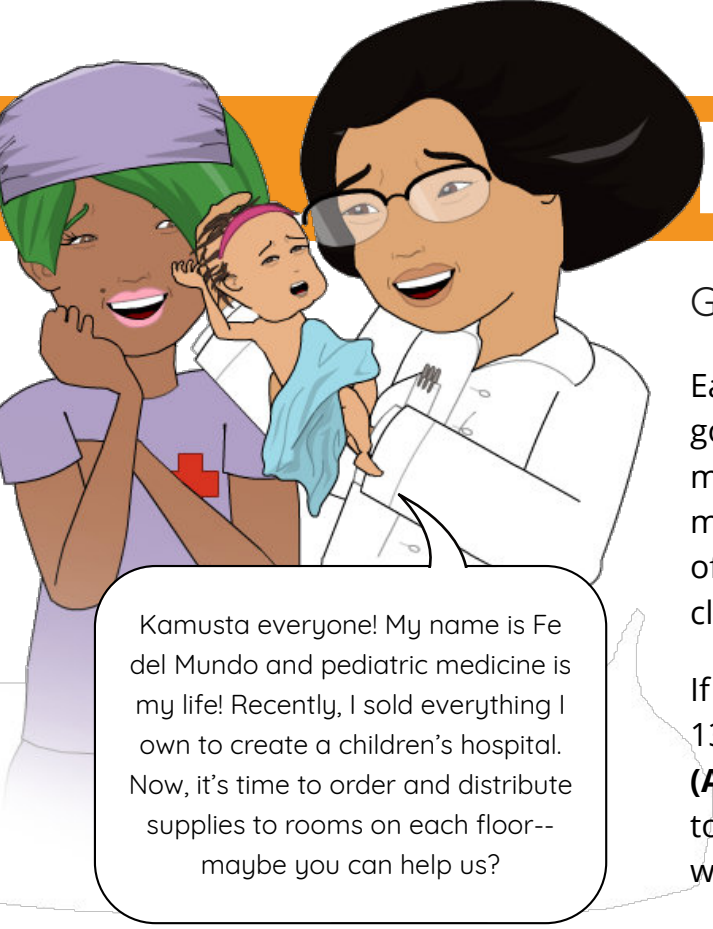


DIVISIBILITY CONTINUED

GAIM Newsletter Activity for **Vol. 1 | Issue 2**

Each of the 5 floors of the Fe Del Mundo Medical Center goes through about 9 bottles of hand sanitizer every month. The hospital orders supplies only one time per month, and Fe would prefer to buy enough to stock each of the five floors' supply closets equally so that each closet always has the same amount on hand.

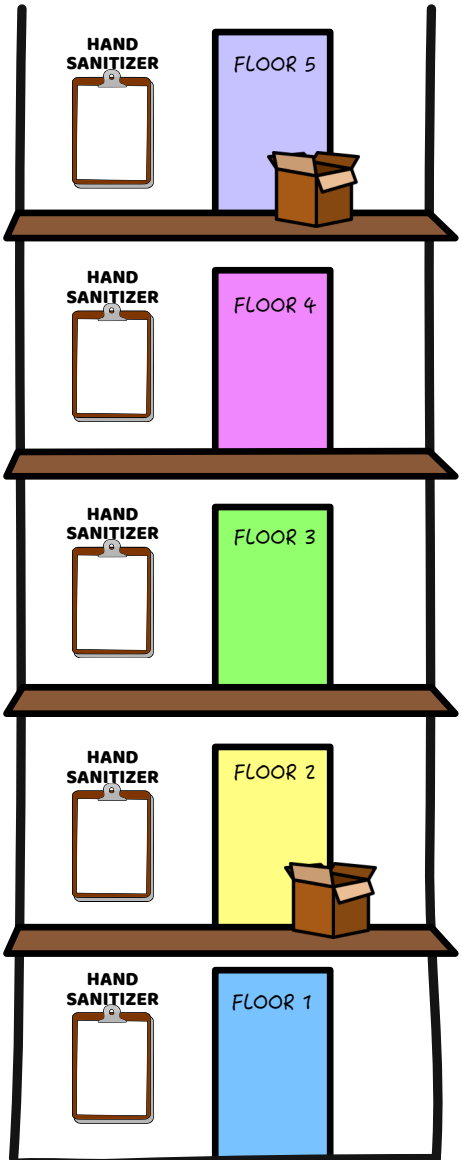
If the factory only sells hand sanitizer in cartons of 72, 135, or 195 bottles, which carton size should she order? **(Answer 1A in 'Solutions')** After how long will she need to order hand sanitizer again? **(1B)** In today's activity, we'll learn how to answer this question at a glance!

BEFORE WE BEGIN

In today's activity, we are going to build on what we've learned about divisibility by 2, 4, and 8 by developing rules of divisibility for 3, 9, 5, 10, and 6.

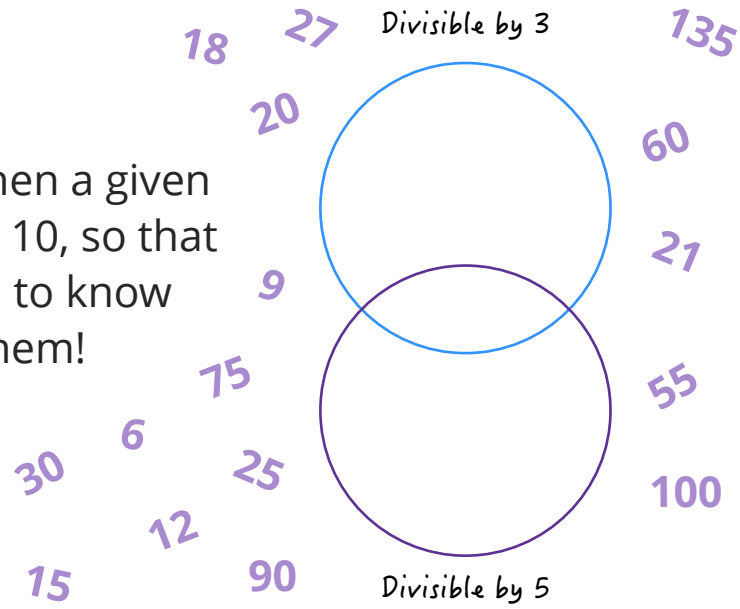
Let's first go over some of the concepts that we reviewed in the 2, 4, 8 activity. Make sure you understand them one at a time, as they build on each other:

- An **integer** is a whole number that can be positive, negative, or 0.
- One integer is **divisible** by another when you divide the first by the second and the result is an integer.
- An integer greater than 1 is **prime** when it is divisible by exactly two integers: 1 and itself.
- A number is **composite** when it is divisible by more than 1 and itself.
- Any integer greater than 1 is either a prime number, or can be formed by multiplying prime numbers together.
- If two numbers are divisible by another number, then their sum is divisible by that number.



GOAL:

> Develop the divisibility rules for when a given integer is divisible by 3, 9, 5, 6, and 10, so that we don't have to do lots of division to know whether a number is divisible by them!



Prime Factorization Review

FACTOR TREES

Let's make some more factor trees! Divisibility is about numbers making other numbers, and so is factorization. Create factor trees by dividing the numbers up into smaller numbers. Each set of smaller numbers that you create should multiply together to create the number above it.

TRY IT YOURSELF!

Instructions: Can you complete the factor trees for the numbers below? Pay attention to the prime factors. Do you notice any trends? You can now say that each of the top numbers is "divisible" by the bottom numbers!

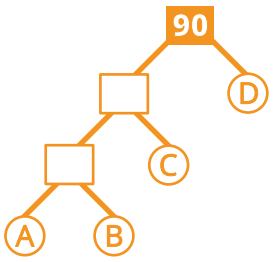
= prime number = composite number

#T1-A 135

#T1-B 135

#T1-C 135

#T1-D 135



#T2 CHALLENGE

If the symbol "≤" means **is less than or equal to,** complete the T2 tree so that A ≤ B ≤ C ≤ D.

#T2 90

DIVISIBILITY RULES

3, 9, 5, 10, 6

RULE FOR #3

A number is divisible by 3 **if the sum of its digits is divisible by 3**.

Which of the following numbers are divisible by 3? (Circle your answer.)

642 **741** **428416**

WHY?

Any number XYZ can be written as (X x **100**) + (Y x **10**) + Z**.** Let's take 642 as an example. Since **100** = **1** + **99** and **10** = **1** + **9,** we can rewrite the above formula this way:

(6 x (1+99)) + (4 x (1+9)) + 2

6 (1) + 6 (99) + 4(1) + 4 (9) + 2

Now, let's reorder these logically: 6 + 4 + 2 + 6(99) + 4(9)**.**

12 + 6(99) + 4(9) - All three terms are divisible by 3, so 642 is divisible by 3.

Let's go over the other examples:

741
XYZ

(7 x **100**) + (4 x **10**) + 1

(7 x (**99** + **1**)) + (4 x (**9** + **1**)) + 1

7(99) + 7(1) + 4(9) + 4(1) + 1

7 + 4 + 1 +7(99) + 4(9)

12 + 7(99) + 4(9)

both divisible by 3, so their sum is divisible by 3

12 is divisible by 3, so 741 is divisible by 3.

428416
XYZXYZ

4 ~~x 100000~~ → (1 + 99999)
+ 2 ~~x 10000~~ → (1 + 9999)
+ 8 ~~x 1000~~ → (1 + 999)
+ 4 ~~x 100~~ → (1 + 99)
+ 1 ~~x 10~~ → (1 + 9)
+ 6

99999, 9999, 999, 99, & 9 can all be removed, since we know that they are divisible by 3.

4 + 2 + 8 + 4 + 1 + 6 = 25

25 is not divisible by 3,
so 428416 is **not** divisible by 3.

RULE FOR #9

A number is divisible by 9 **if the sum of its digits is divisible by 9.**

Which of the following numbers are divisible by 9? (Circle your answer.)

642 **3465** **12723**

WHY?

Let's try a similar strategy to the one we used to determine divisibility by 3. **ABCD** can be written as (**A** x **1000**) + (**B** x **100**) + (**C** x **10**) + **D.** Let's take **3465** as an example.

Before we tackle **3465**, let's rewrite those multipliers like we did in the previous examples.

$$(3 \times 1000) + (4 \times 100) + (6 \times 10) + 5$$

$$(3 \times (999 + 1)) + (4 \times (99 + 1)) + (6 \times (9 + 1)) + 5$$

$$3(999) + 3(1) + 4(99) + 4(1) + 6(9) + 6(1) + 5$$

$$3 + 4 + 6 + 5 + 3(999) + 4(99) + 6(9)$$

$$18 + 3(999) + 4(99) + 6(9)$$

all divisible by 9,
so their sum is divisible by 9

18 is divisible by **9**, so **3465** is divisible by **9**.

What about **12723**?

$$(1 \times 1000) + (2 \times 1000) + (7 \times 100) + (2 \times 10) + 3$$

$$(1 \times (9999 + 1)) + (2 \times (999 + 1)) + (7 \times (99 + 1)) + (2 \times (9 + 1)) + 3$$

$$1(9999) + 1(1) + 2(999) + 2(1) + 7(99) + 7(1) + 2(9) + 2(1) + 3$$

$$1 + 2 + 7 + 2 + 3 + 2(999) + 7(99) + 2(9)$$

$$15 + 2(999) + 7(99) + 2(9)$$

all divisible by 9,
so their sum is divisible by 9

15 is **NOT** divisible by **9**, so **12723** is **NOT** divisible by **9**.

RULE FOR #5

A number is divisible by 5 **if the last digit is a 0 or 5.** This is the same as saying:

A number is divisible by 5 if the last digit is divisible by 5.

Which of the following numbers are divisible by 8? (Circle your answer.)

642 **1495** **428410**

WHY?

AB can be written as **A** x **10** + **B.** Let's take the above numbers as examples.

642	**1495**	**428410**
AB	**AB**	**AB**
64 x 10 + **2**	**149** x 10 + **5**	**42841** x 10 + **0**
640 is divisible by 5, 2 is not, so	**1490 is divisible by 5, 5 is also divisible by 5, so**	**428410 is divisible by 5, 10 is also divisible by 5, so**
642 is not divisible by 5.	**1495 is divisible by 5.**	**428410 is divisible by 5.**

Because **10** is divisible by **5**, any number that is a multiple of **10** is divisible by **5**. Likewise, **5** is divisible by **5** and so the sum of any numbers divisible by **5** is divisible by **5**. This boils down to the last digit being divisible by **5**. And the only two digits divisible by **5** are **0** and **5**.

TRY IT YOURSELF!

Word Problem: Shirley is sorting **45** trolls into at least 2 bins at the toy store. There must be at least 2 trolls in each bin. She needs to put the same number of trolls in each bin without any leftover trolls. How many bins could Amelia use for her trolls? (There are multiple answers-- try and come up with as many as you can!) **(2)**

RULE FOR #10

A number is divisible by 10 **if the last digit is a 0.**

Which of the following numbers are divisible by 10? (Circle your answer.)

642 **1240** **428410**

WHY?

XY can be written as **X** x **10** + **Y.** Let's take **1240** as an example.

Before we tackle **1240**, let's rewrite those multipliers like we did in the previous examples.

124 x **10** + **0**

Since **0** and any multiple of **10** are divisible by **10**, the sum of the two will be divisible by **10**. Therefore, **1240** is divisible by 10.

642

64 x **10** + **2**

640 is divisible by 10, 2 is not, so

642 is not divisible by 10.

428410

42841 x **10** + **0**

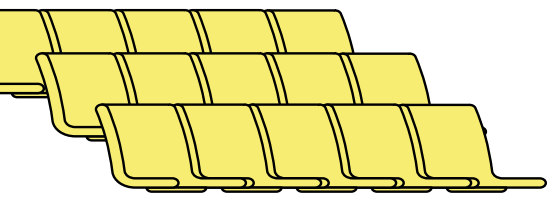
428410 is divisible by 10, 0 is also divisible by 10, so

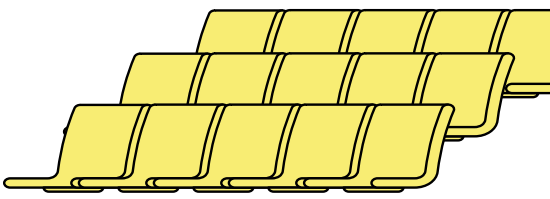
428410 is divisible by 10.

Since **10** is divisible by **10**, any multiple is divisible by **10.** If the last digit is a **0**, then the number is divisible by **10**.

TRY IT YOURSELF!

Word Problem: Lina Bo Bardi is helping Fe design an operating theatre in the hospital that will have 150 seats. She wants to divide the seats into at least 3 equal sections but no more than 6 equal sections. How many different numbers of sections could Lina use? How many rows could there be? How many seats could be in each row? *There are multiple answers! Get creative!* **(3)**





RULE FOR #6

A number is divisible by 6 **if the number is divisible by 2 and divisible by 3.** Or, a number is divisible by 6 **if the number is even and the sum of its digits is divisible by 3.**

Which of the following numbers are divisible by 6? (Circle your answer.)

642 **1495** **428410**

WHY?

A number is divisible by **6** if the number is divisible by **2** and divisible by **3** since **6** = **2** x **3,** or, we can state the Rule for **6** in terms of the Rules for **2** and **3**. That is, a number is divisible by **6** if the number is even and the sum of its digits is divisible by **3**.

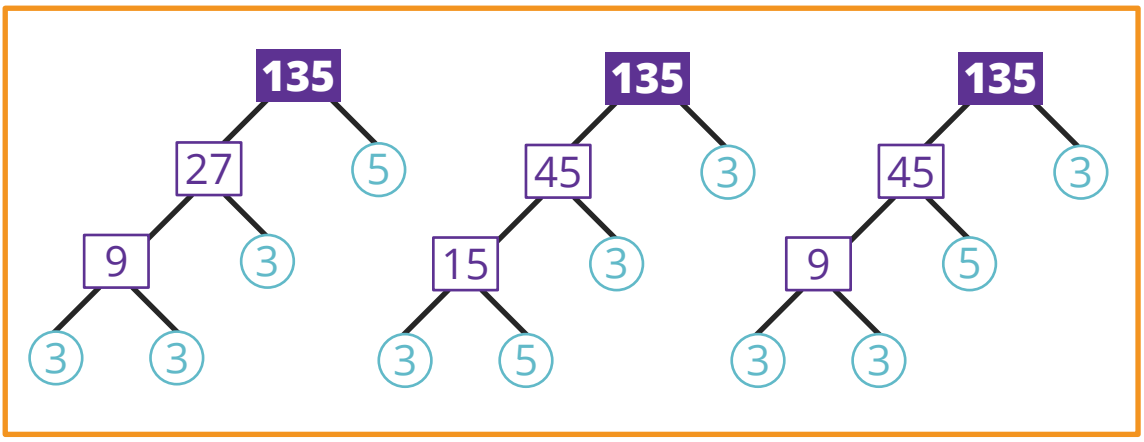
	642	**1495**	**428410**
2	**642 is even, therefore it is divisible by 2.**	**1495 is not even, therefore it is not divisible by 2 (or 6).**	**428410 is even, therefore it is divisible by 2.**
3	**6 + 4 + 2** **12** **12 is divisible by 3.**	**1 + 4 + 9 + 5** **19** **19 is not divisible by 3, therefore 1495 is not divisible by 6.**	**4 + 2 + 8 + 4 + 1 + 0** **19** **19 is not divisible by 3, therefore 428410 is not divisible by 6.**
6	✔	✘	✘

TRY IT YOURSELF!

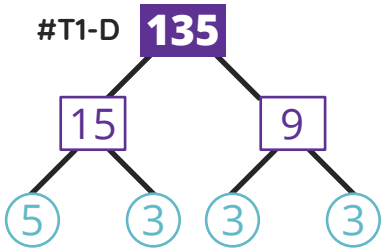
Word Problem: Fe del Mundo brings 6 balloons to each child that has a birthday while they are in treatment. The hospital purchases balloons and tanks of helium separately. If each tank of helium can fill 54 balloons, and balloons come in packs of either 30, 60, or 108, what is the smallest number of helium tanks and balloons the hospital should purchase to ensure there are no balloons or helium left over? How many birthdays can they accommodate with their purchase? **(4)**

FACTOR TREES SOLUTIONS

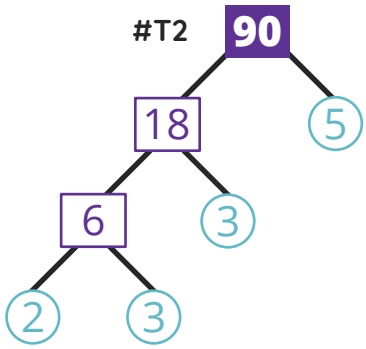
#T1-A THROUGH #T1-C



#T1-D



#T2



ACTIVITY SOLUTIONS

1A. 135

1B. 3 months

2. Possible numbers of bins = 3, 5, 9, 15

3.

Sections	Rows	Seats in Each Row
3	5	10
3	10	5
3	2	25
3	25	2
5	3	10
5	10	3
5	5	6
5	6	5
5	2	15
5	15	2
6	5	5

4. 2 tanks of helium, 1 pack of 108 balloons, 18 birthdays

Divisibility by 3, 5, 6, 9, & 10

QUICK REVIEW

Place a checkmark in the box if the top number is divisible by the number next to the box.

Q1) **21**

- by 3 ☐
- by 5 ☐
- by 6 ☐
- by 9 ☐
- by 10 ☐

Q2) **189**

- by 3 ☐
- by 5 ☐
- by 6 ☐
- by 9 ☐
- by 10 ☐

Q3) **1441**

- by 3 ☐
- by 5 ☐
- by 6 ☐
- by 9 ☐
- by 10 ☐

Q4) **111000**

- by 3 ☐
- by 5 ☐
- by 6 ☐
- by 9 ☐
- by 10 ☐

Complete the following word problems. If the words are confusing you, focus first on the numbers. You may find it easier to check all of the numbers' divisibility first!

Problem #WP1

A. Company "Three, Five, Nine, Ten and Co." has 225 employees. The same number of employees work on each shift. How many shifts are possible from the choices below? Please circle all that apply.

Three Five Six Nine Ten

B. What is the smallest number of employees they would need to hire in addition to the original 225 employees so that all of the choices above would be possible?

Problem #WP2

In the Small Town of "ThreeFiveNineTenville", the mayor hosted a charity fundraiser for all 1,860 residents and every resident attended. Each row of tables has the same number of tables and each table sits exactly 10 people. How many rows of tables are possible from the following choices assuming that every table was filled.

Three Five Six Nine Ten

QUICK REVIEW ANSWERS

Q1) **21**

- by 3 ■
- by 5 ☐
- by 6 ☐
- by 9 ☐
- by 10 ☐

Q2) **189**

- by 3 ■
- by 5 ☐
- by 6 ☐
- by 9 ■
- by 10 ☐

Q3) **1441**

- by 3 ☐
- by 5 ☐
- by 6 ☐
- by 9 ☐
- by 10 ☐

Q4) **111000**

- by 3 ■
- by 5 ■
- by 6 ■
- by 9 ☐
- by 10 ■

WORD PROBLEM ANSWERS

WP1-A. 3, 5, and 9
WP1-B. 45 employees
WP2. 3 or 6