GAIM 2019

Round 1 Solutions
Elementary Division

IMMORTALS: Storm

QUESTION

Three soldiers report the rumors that they heard:
Soldier 1: “The storm will be in the north. It will be at 9am.”
Soldier 2: “The storm will be in the east. It will be at 10am.”
Soldier 3: “The storm will be in the east. It will be at 11am.”
If each soldier said at least one true statement, when and where will the storm be?

ANSWER

East, 9AM

SOLUTION

We have two facts each and three soldiers. For each soldier to have made one true claim, one of the true claims must be something that two soldiers agree on. The only repeated claim is that the storm will be in the east, so that must be true.

Let’s check. Soldier 2 and Soldier 3 both say that the storm is in the east. Let’s pretend they are both wrong about where the storm is. Since they disagree on when the storm will occur, one of them must also be wrong about when the storm will be. This unlucky soldier will be wrong about both where and when the storm is, which we know is not possible!

Here’s what the truth table would look like if the storm were not in the east, and Soldier 2 were right about the time.

	Statement 1: Where is the storm?	**Statement 2: When will the storm be?**
Soldier 1		
Soldier 2	**False**	**True**
Soldier 3	~~**False**~~	~~**False**~~

Since Soldier 2 and Soldier 3 are correct about the storm's location, the storm must be in the east, not the north. Therefore, Soldier 1 must be wrong about the location, so he must be right about the time.

Truth Table

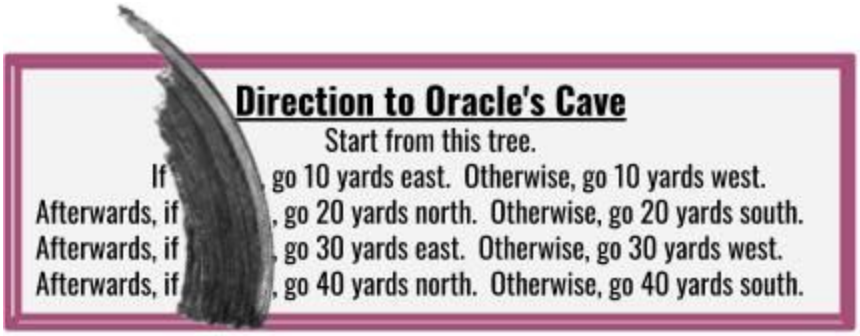
	Statement 1: Where is the storm?	**Statement 2: When will the storm be?**
Soldier 1	False	**True**
Soldier 2	True	False
Soldier 3	True	False

Therefore, The storm will be in the East at 9AM.

IMMORTALS: Oracle

QUESTION

Gharghâvol the Clumsy Bird erases either “10 is prime” or “10 is composite” from each line of directions to Yolo the Oracle’s Cave. The directions now read:



(1) Start from this tree.
(2) If <smudged>, go 10 yards east. Otherwise, go 10 yards west.
(3) Afterwards, if <smudged>, go 20 yards north. Otherwise, go 20 yards south.
(4) Afterwards, if <smudged>, go 30 yards east. Otherwise, go 30 yards west.
(5) Afterwards, if <smudged>, go 40 yards north. Otherwise, go 40 yards south.

The directions lead to Yolo’s Cave. How many different possible locations are there for Yolo’s Cave?

ANSWER:

16 different possible locations.

SOLUTION

The east-west paths have 4 different possibilities:

10E + 30E = 40 East
10E + 30W = 20 West
10W + 30E = 20 East
10W + 30W = 40 West

Similarly, the north-south paths have 4 different possibilities:

20N + 40N = 60 North
20N + 40S = 20 South
20S + 40N = 20 North
20S + 40S = 60 South

For each east-west possibility there are 4 unique north-south possibilities, therefore there are 4 * 4 = 16 possible locations.

IMMORTALS: YOLO

QUESTION

Yolo the Oracle is on vacation and leaves directions from her Cave to her Treehouse.

(1) If 39 is prime, go 30 yards east. Otherwise, go 30 yards west.
(2) Afterwards, if 100 is a perfect square, go 40 yards north. Otherwise, go 40 yards south.
(3) Afterwards, if 123,456,789 is divisible by 9, go 30 yards east. Otherwise, go 30 yards west.
(4) Afterwards, if 116,688 is divisible by 11, go 40 yards north. Otherwise, go 40 yards south. You have now arrived at my Treehouse!

What is the distance, in yards, between Yolo's Cave and her Treehouse?

ANSWER:

80 yards.

SOLUTION

Let's take a look at the statements in order.

Statement 1: If 39 is prime, go 30 yards east. Otherwise, go 30 yards west.
39 = 3 * 13. Therefore, it is not prime. We go 30 yards west.

Statement 2: Afterwards, if 100 is a perfect square, go 40 yards north. Otherwise, go 40 yards south.
100 = 10 * 10 is a perfect square. We go 40 yards north.

Statement 3: Afterwards, if 123,456,789 is divisible by 9, go 30 yards east. Otherwise, go 30 yards west.

There are two ways of finding whether 123,456,789 is divisible by 9.

Method 1: Divide 123,456,789 by 9 and check that the remainder is zero.

123,456,789 ÷ 9 = 13,717,421 with remainder 0.

Method 2: Use the divisibility rule for 9.

We check whether 123,456,789 is divisible by 9 by summing its digits:

$$1+2+3+4+5+6+7+8+9 = 45$$

Since 45 is divisible by 9, the number 123,456,789 is divisible by 9.

We go 30 yds east.

<u>Statement 4</u>: Afterwards, if 116,688 is divisible by 11, go 40 yards north. Otherwise, go 40 yards south. You have now arrived at my Treehouse!

There are two ways of finding whether 116,688 is divisible by 11.

Method 1: Divide 116,688 by 11 and check that the remainder is zero.

116,688 ÷ 11 = 10,608 with remainder 0.

Method 2: Use the divisibility rule for 11.

What is the alternating sum of the digits of 116,688?

$$-1+1-6+6-8+8 = 0$$

Since 0 is divisible by 11, go 40 yards north.

Finally, we combine the instructions:

1. 30 yards west
2. 40 yards north
3. 30 yards east
4. 40 yards north

Now, the 30 yards east and the 30 yards west cancel, so the YOLO's treehouse is 80 yards north of his cave. The total distance is 80 yards.

FREEDOM: Mbundu

QUESTION

Nzinga is buying the book "History of the Mbundu" for her local school. Paperback copies cost $1.50 and hardcover copies cost $4. If she spends at least $50, she gets a free teacher's edition.

Nzinga has exactly $50. What is the largest number of copies of "History of the Mbundu" that she can buy so that she also gets a free teacher's edition?

ANSWER

30 copies.

SOLUTION

Paperback copies are cheaper than hardcover copies. If Nzinga would like to buy as many books as possible, she should buy as many paperback copies as possible.

With $50, Nzinga can buy at most 33 paperback books. This would cost $49.50, less than the $50 she needs to spend to receive the free teacher's edition.

Let's try switching some paperback copies for hardback copies. If she buys three fewer paperbacks (3 x $1.50 = $4.50) and and one more hardback (1 x $4.00 = $4.00), she will spend $0.50 less.

Instead of buying 0 hardbacks and 33 paperbacks, if Nzinga buys 1 hardback and buys 30 paperbacks, her total cost will be $49.00. Doing this switch again will lead to 27 paperbacks and 2 hardbacks for a total of $48.50.

Now, Nzinga will have exactly $1.50 leftover. This is enough money for one more paperback. So she can get 28 paperbacks and 2 hardbacks to spend $50 and get her "free" teacher's edition.

Therefore, she buys 28 + 2 = 30 copies in total.

# paperback copies	# hardcover copies	Total Cost	Change if Nzinga pays with $50
33	0	(33 x $1.50) + (0 x $4.00) = $49.50	$50 - $49.50 = $0.50
30	1	(30 x $1.50) + (1 x $4.00) = $49.00	$50 - $49.00 = $1.00
27	2	(27 x $1.50) + (2 x $4.00) = $48.50	$50 - $48.50 = $1.50
28	2	(28 x $1.50) + (2 x $4.00) = $50.00	$50 - $50 = $0 😎

FREEDOM: Kings

QUESTION

What is the smallest number of chess kings Nzinga can place on an 8 by 8 chessboard in such a way that no additional kings can be placed without two of the kings attacking each other?

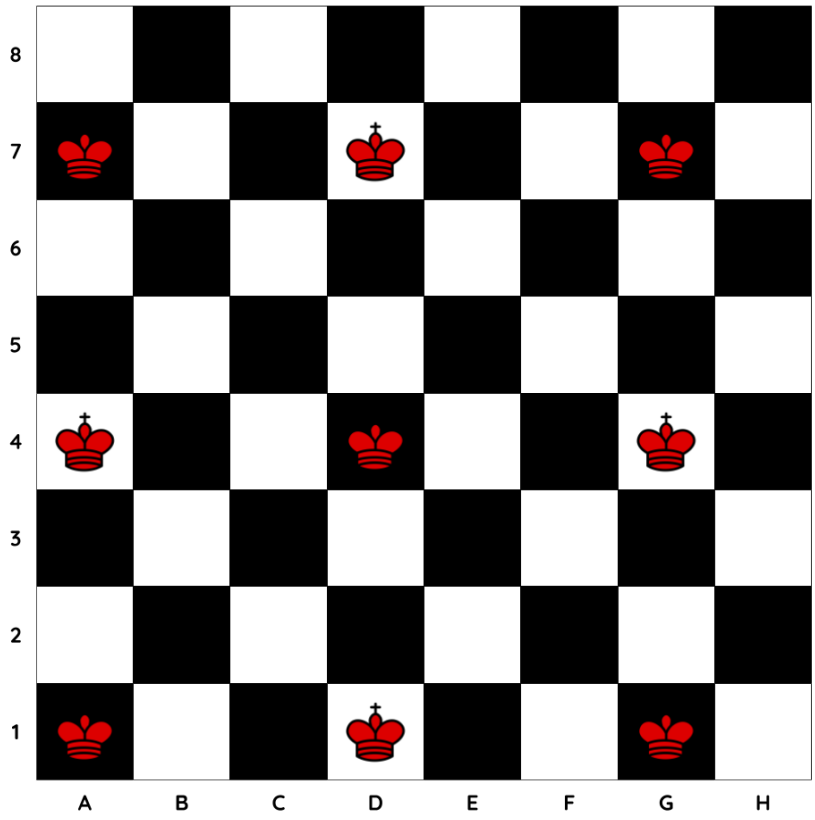
(Please note the diagram shows a 4 by 4 chessboard.)

ANSWER

9 kings

SOLUTION:

Label the rows of the chessboard, in order, 1 through 8, and the columns, in order with the letters, a through h. Now, we can place 9 kings on the squares A1, A4, A7, D1, D4, D7, G1, G4, and G7 without any of them attacking each other and it is impossible to place an additional king without attacking of these kings. (See diagram below.)



Is it possible to use fewer than 9 kings?

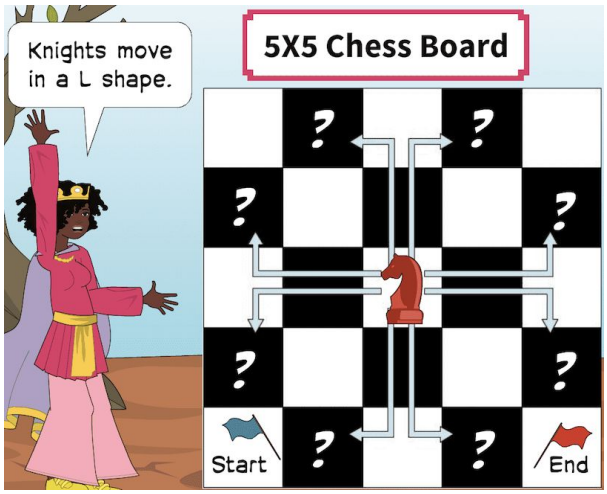
Let's consider the spaces that the kings are currently placed on (A1, A4, A7, D1, D4, D7, G1, G4, and G7). One king cannot attack two of these nine spaces at the same time. Said another way, if we have eight or fewer kings, we cannot control all nine spaces. Therefore, we need at least nine kings to control the board.

FREEDOM: Knights

QUESTION

What is the smallest number of moves a knight would have to make to travel from one corner to a neighboring corner on an 11 by 11 chessboard?

(Please note the diagram shows a 5 by 5 chessboard.)



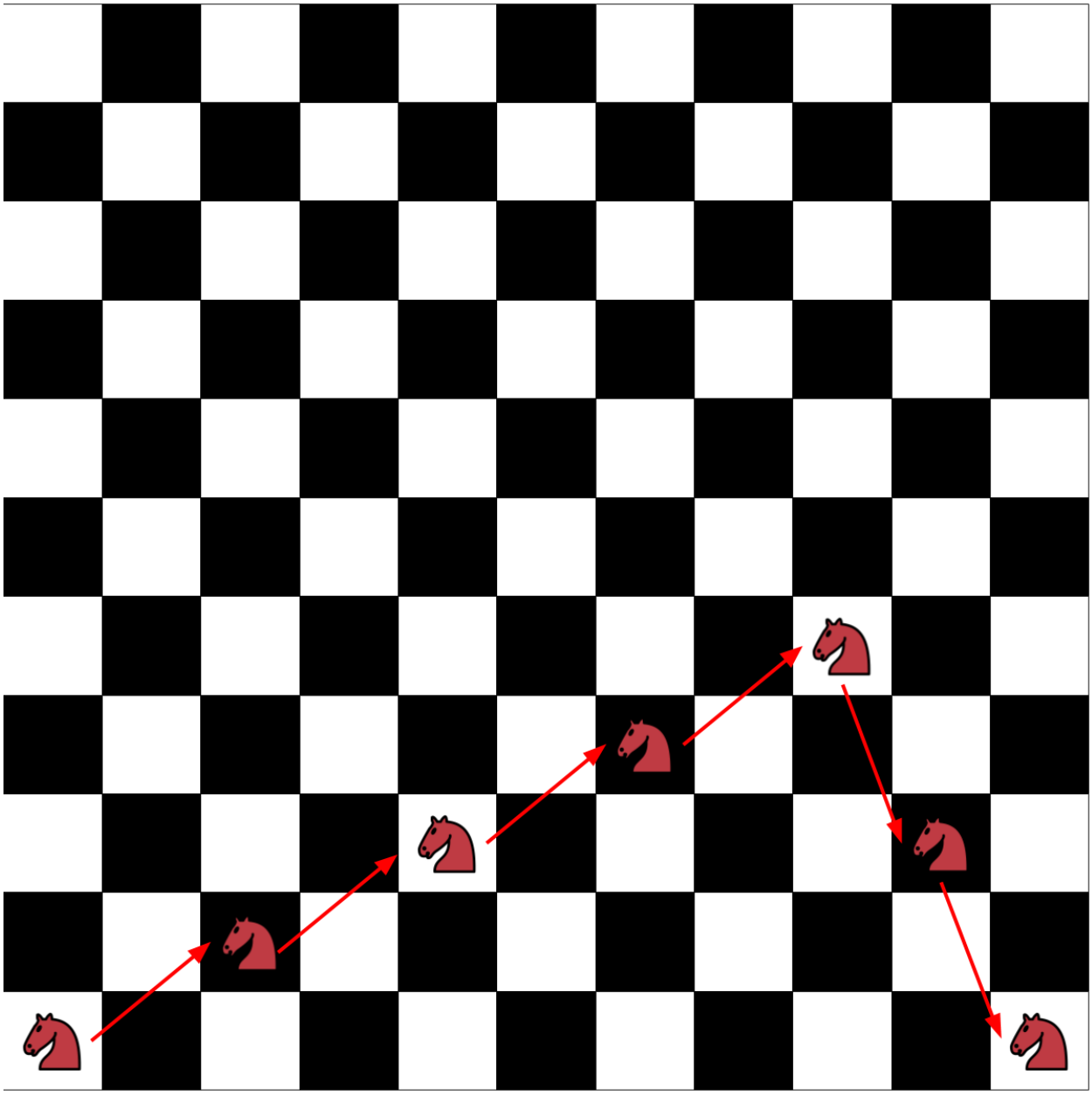
ANSWER

6 moves.

SOLUTION:

First, we note that on an 11 x 11 board, all the corner squares are of the same color. On each move a knight changes the color of the square it rests on. To get to a neighboring corner, a knight will need to travel 10 spaces using an even number of moves.

Now, four moves is insufficient. In four moves, a knight can move at most 8 spaces in any direction and it needs to move 10 spaces horizontally or vertically. Six moves is enough. A knight can travel from the lower left to lower right corners, for instance, by moving 2-right-and-1-up four times, and then 1-right-and-2-down twice. See diagram below.



RADIUM: Leaf

QUESTION

One gold block can make a single gold leaf and a heap of leftover shavings. After Marie makes six gold leaves, she melts the collected shavings into a new gold block.

How many gold leaves can Marie make out of 216 gold blocks?

ANSWER

259 gold leaves.

SOLUTION:

The 216 gold blocks can produce 216 gold leaves plus enough shavings for 216 ÷ 6 = 36 new gold blocks.

These 36 gold blocks are enough to produce 36 more gold leaves plus enough shavings for 36 ÷ 6 = 6 new gold blocks.

These 6 new gold blocks are enough to produce 6 gold leaves plus shavings for 6 ÷ 6 = 1 new gold blocks.

This final gold block can produce 1 final gold leaf.

In total, Marie can make 216 + 36 + 6 + 1 = 259 gold leaves.

RADIUM: Poison

QUESTION

Potion A is poison. Potion B is water. Potion C is an antidote to Potion A. There is an unlimited supply of each potion.

Shirley takes three sips in a row, not necessarily of different potions. If she takes at least one sip of the poison, she will be poisoned unless she also drinks the antidote. If Shirley drinks any amount of antidote at any time, it will protect her from even a double dose of poison. How many different drink orders result in her not being poisoned at the end of her three sips?

(For example, antidote, poison, poison and poison, poison, antidote are both drink orders that do not poison Shirley.)

ANSWER

20 drink orders.

SOLUTION:

Solution 1:

We can directly count the number of safe drink orders.

Let's make a map of all of the drink orders.

After the first sip, Shirley's drink orders look like:

First Sip		
A	B	C

After the second sip, Shirley's drink orders look like:

		First Sip		
		A	B	C
Second Sip	A	AA	BA	CA
	B	AB	BB	CB
	C	AC	BC	CC

After the third sip, Shirley's drink orders look like:

		First Sip		
		A	B	C
Second Sip	A	AAA AAB AAC	BAA BAB BAC	CAA CAB CAC
	B	ABA ABB ABC	BBA BBB BBC	CBA CBB CBC
	C	ACA ACB ACC	BCA BCB BCC	CCA CCB CCC

Any combinations that include Potion C are safe orders. Therefore, the 19 drink orders highlighted below are safe.

		First Sip		
		A	B	C
Second Sip	A	AAA AAB AAC	BAA BAB BAC	CAA CAB CAC
	B	ABA ABB ABC	BBA BBB BBC	CBA CBB CBC
	C	ACA ACB ACC	BCA BCB BCC	CCA CCB CCC

If a drink order does not include Potion C, it is safe if it also does not include Potion B. There is only one drink order that does not include potions B or C, highlighted in orange below.

		First Sip		
		A	B	C
Second Sip	A	AAA AAB AAC	BAA BAB BAC	CAA CAB CAC
	B	ABA ABB ABC	BBA BBB BBC	CBA CBB CBC
	C	ACA ACB ACC	BCA BCB BCC	CCA CCB CCC

Therefore, there are 19 + 1 = 20 safe drink orders.

Solution 2:

Any drink orders that are not poisonous are safe. We can subtract the number of poisonous drink orders from the total number of drink orders. A drink order is poisonous when it includes Potion B (poison) but does not include Potion C (antidote).

Exactly 2 × 2 × 2 = 8 drink orders do not include antidote, highlighted in pink below.

		First Sip		
		A	B	C
Second Sip	A	AAA AAB AAC	BAA BAB BAC	CAA CAB CAC
	B	ABA ABB ABC	BBA BBB BBC	CBA CBB CBC
	C	ACA ACB ACC	BCA BCB BCC	CCA CCB CCC

One of those eight drink orders contains all water and does not contain poison.

		First Sip		
		A	B	C
Second Sip	A	AAA AAB AAC	BAA BAB BAC	CAA CAB CAC
	B	ABA ABB ABC	BBA BBB BBC	CBA CBB CBC
	C	ACA ACB ACC	BCA BCB BCC	CCA CCB CCC

Therefore, 8 - 1 = 7 of these drink orders are poisonous. All other drink orders are safe. Therefore, 27 - 7 = 20 drink orders are safe.

RADIUM: Supreme

QUESTION

Irene sells copper chain links at her machinery store. Each chain of 10 links costs $10. Irene charges an additional $1 to either split a chain at a certain spot or join two chains of links together. Marie wants to make four chemistry bracelets for her students. The four bracelets will have 7, 9, 11, and 13 links. What is the minimum amount of money that Marie needs?

ANSWER

$44

SOLUTION:

Irene can do this by following the below steps.

Step	**Cost**
Buy 4 links of length 10.	4 x $10 = $40
Break the first link into lengths 9 and 1.	$1
Attach the link of length 1 to a second link of length 10.	$1
Break the third link into lengths 7 and 3.	$1
Attach the link of length 3 to the fourth link of length 10.	$1
Total Cost:	**$44**

ORBIT: Flag

QUESTION

Shirley is sewing an American flag to bring on the next space mission.

The flag has a total of 13 stripes, alternating between red and white, and a blue rectangle with 50 white stars. It takes 5 minutes to stitch any two stripes together and a half hour to sew on the star-studded rectangle. How many minutes would it take Shirley to make the flag?

ANSWER

90 minutes

SOLUTION:

To sew together 13 stripes, Shirley will need to sew 12 seams in between the stripes. This will require 12 x 5 minutes = 60 minutes.

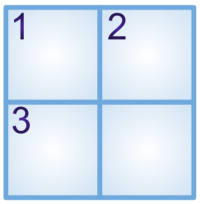
Sewing on the stars rectangle will take an additional 30 minutes.

In total, Shirley sews for 60 minutes + 30 minutes = 90 minutes.

ORBIT: Collision

QUESTION

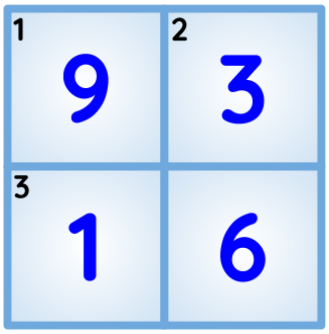
Fill out the above crossword puzzle:



(1 Across) A composite number, greater than the answer to (1 Down).
(3 Across) A perfect square.
(1 Down) A composite number, smaller than the answer to (1 Across).
(2 Down) A perfect square.

(Note: Numbers cannot start with 0.)

ANSWER



SOLUTION:

2-Down and 3-Across are both two-digit perfect squares. That means they must be among 16, 25, 36, 49, 64, and 81. They cannot be the same because if they are, then 1-Down and 1-Across would be the same number, and they are not. Since they share their ones digit, they must be 36 and 16. Which one is which?

1-Across and 1-Down are composite numbers that share their tens digit. 1-Across is greater than 1-Down, so the second digit of 1-Across must be greater than that of 1-Down.

This means that 2-Down must be larger than 3-Across, so 2-Down is 36 and 3-Across is 16.

Now, we just need to find the number in the top left corner. We know that 1-Across and 1-Down are composite. Let's try different digits in the top left corner!

Top Left	**1-Down**	**1-Across**	**Are 1-down and 1-Across both composite?**
1	11	13	No. 11 and 13 are both prime.
2	21	23	No. 23 is prime.
3	31	33	No. 31 is prime.
4	41	43	No. 41 and 43 are both prime.
5	51	53	No. 53 is prime.
6	61	63	No. 61 is prime.
7	71	73	No. 71 and 73 are both prime.
8	81	83	No. 83 is prime.
9	91	93	Yes! 91 = 7 x 13 and 93 = 3 x 31.

ORBIT: Jumping Jupiter

QUESTION

How many whole numbers between 1 and 100 satisfy all three of the following conditions?

(A) One of its digits is 3, or it is divisible by 3.
(B) One of its digits is 5, or it is divisible by 5.
(C) One of its digits is 7, or it is divisible by 7.

ANSWER

3 numbers.

SOLUTION:

Let's start with the condition C, because 7 has the fewest multiples less than or equal to 100. We create two lists:

1. 7 as a digit: 7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 87, and 97
2. Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, and 98.

Now, let's narrow down the two lists above by condition B.

1. Of numbers that include 7 as a digit, only 57, 70, and 75 either have 5 as a digit or are a multiple of 5.
2. Of numbers that are multiples of 7, only 35, 56, and 70 either have 5 as a digit or are a multiple of 5.

Now let's take a look at condition C.

1. 35 has 3 as a digit.
2. 57 and 75 are multiples of 3.

Neither 70 nor 56 satisfies either parts of condition 1.

Only 35, 57, and 75 satisfy these all three conditions, so the answer is 3 numbers.

In the diagram below,

- Numbers in bold satisfy condition A

- Numbers highlighted in green satisfy condition B
- Numbers in blue satisfy condition C
- Numbers with a smiley face satisfy all three conditions.

1	2	**3**	4	5	**6**	7	8	**9**	10
11	**12**	**13**	14	**15**	16	17	**18**	19	20
21	22	**23**	**24**	25	26	**27**	28	29	**30**
31	**32**	**33**	**34**	**35** 😎	**36**	**37**	**38**	**39**	40
41	**42**	**43**	44	**45**	46	47	**48**	49	50
51	52	**53**	**54**	55	56	**57** 😎	58	59	**60**
61	62	**63**	64	65	**66**	67	68	**69**	70
71	**72**	**73**	74	**75** 😎	76	77	**78**	79	80
81	82	**83**	**84**	85	86	**87**	88	89	**90**
91	92	**93**	94	95	**96**	97	98	**99**	100

DIAMOND: The Spinster

QUESTION

When written with calculator digits (shown above), how many two-digit numbers remain two-digit numbers when rotated by 180 degrees?

(Remember, numbers cannot start with 0.)

ANSWER

36 numbers.

SOLUTION:

The digits 1, 2, 5, 6, 8, 9, and 0 will remain digits when rotated. Any of these digits can be used to form a two-digit number except 0. Why?

The tens digit can not be 0 as two-digit numbers can not start with 0. The ones digit can not be 0 either, because it would become a 0 in the tens digit when the two-digit number is rotated.

We have 6 choices for the tens digit, and 6 choices for the ones digit. 6 * 6 = 36 possibilities for the two-digit number.

DIAMOND: Princess & The Frog

QUESTION

A princess and a frog are having an argument. The princess is yelling at her top speed, 6 words per second. The frog slips in one word immediately after every five of her words. Every time he speaks, he calms the princess's anger and slows her speech by one word per second. To the nearest second, how long does it take him to render her speechless and bring peace back to the kingdom?

(They are having a very respectful argument--the frog never speaks over the princess!)

ANSWER

12 seconds

SOLUTION:

The Princess's first 5 words are at a pace of 6 words per second, her next 5 are at a pace 5 words per second, her next 5 are at a pace of 4 words per second, then her next 5 are 3 words per second, then 5 at 2 words per second, and finally 5 at 1 word per second before she is rendered mute.

Her total talking time, in seconds, before being rendered mute is:

$$5/6 + 5/5 + 5/4 + 5/3 + 5/2 + 5/1$$
$$= 5/6 + 5 + 5/4 + 5/3 + 5/2 + 1$$

Let's combine the whole number parts and the fractional parts:

$$= (5 + 1) + (5/6 + 5/4 + 5/3 + 5/2)$$

We simplify the fractional part by finding the least common denominator.

$$= (6) + (10/12 + 15/12 + 20/12 + 30/12)$$
$$= (6) + (75/12)$$
$$= (6) + (6 + 3/12)$$
$$= 12 + 1/4$$

To the nearest second, it takes the frog 12 seconds.

DIAMOND: Math Whiz

QUESTION

How many prime numbers less than 100 become perfect squares when 2 is added to them?

ANSWER

5 prime numbers.

SOLUTION:

Let's tackle this question by thinking backwards. If a prime number + 2 = a perfect square, then two less than the perfect square will be prime.

Since the prime number is less than 100, the perfect square must be less than 2+100 = 102. The perfect squares less than 102 are:

1, 4, 9, 16, 25, 36, 49, 64, 81, and 100

Two less than each of the perfect squares would have to be prime. That means such a prime would have to be one of -1, 2, 7, 14, 23, 34, 47, 62, 79, and 98.

Of these, 2, 7, 23, 47, and 79 are prime, so there are five primes.

Perfect Square	Perfect Square - 2	Prime or Composite?
1x1 = 1	1-2 = -1	Neither (not positive)
2x2 = 4	4-2 = 2	Prime 😎
3x3 = 9	9-2 = 7	Prime 😎
4x4 = 16	16-2 = 14	Composite
5x5 = 25	25-2 = 23	Prime 😎
6x6=36	36-2 = 34	Composite

7x7=49	49-2 = 47	Prime 😎
8x8=64	64-2 = 62	Composite
9x9=81	81-2 = 79	Prime 😎
10x10=100	100-2 = 98	Composite

DIAMOND: The Best One

QUESTION

Fill in the grid above, satisfying the below conditions:

(1) Each square contains a number from 1 to 4, inclusive.
(2) Each number from 1 to 4 is written in each row exactly once.
(3) Each number from 1 to 4 is written in each column exactly once.
(4) The number written in the top left corner of each region equals either the product or the sum of all the numbers in the region.

8	?	7	
	4	8	
			8
7			

ANSWER

8 4	? 2	7 3	1
2	4 1	8 4	3
1	3	2	8 4
7 3	4	1	2

SOLUTION:

Let's start by labeling all of our squares with letters like this:

8 A	? B	7 C	D
E	4 F	8 G	H
I	J	K	8 L
7 M	N	O	P

Let's start by looking at the region marked 7, containing numbers M and N.

M and N are on the same row, so they must be different digits between 1 and 4. It is impossible for two integers between 1 and 4 to have a product of 7. The only pair of numbers that sum to 7, so M and N are 3 and 4.

Next, let's take a look at the first column on the left. The region containing the numbers A, E, and I is marked 8, so A, E, and I either add or multiply to 8. A, E, I and M are in the same column, so they are different digits from 1 to 4.

If A, E, and I add up to 8, they must be 1, 3, and 4 (1+3+4=8). This is not possible, because M is 3 or 4, and each digit can appear once in each column.

8 A	? B	7 C	D
E	4 F	8 G	H
I	J	K	8 L
7 3	4	O	P

If A, E, and I multiply to 8, they must be 1, 2, and 4 (1 x 2 x 4 = 8). The remaining number in the first column would be 3, so M = 3. This makes N = 4.

The remaining values in the bottom row O and P must be 1 and 2. It's impossible for P to be 1, because L + P = 8 or L x P = 8,. Therefore, P = 2 and O = 1.

8 A	? B	7 C	D
E	4 F	8 G	H
I	J	K	8 L
7 3	4	1	2

We can now fill in the bottom row to get the figure on the right.

Now, as L + 2 = 8 or L * 2 = 8, L must be 4.

And F + J = 4 or F * J = 4 gives us F and J are 1 and 3, The 4 in the second column is already used so they can not be 1 and 4 and they can not be 2 and 2 as they cannot be the same as they are in the same column. So as F and J are 1 and 3, B must be the remaining value in that column, namely 2.

And as the rightmost column now has a 4 and a 2 in it, we can deduce that D and H are 1 and 3. As C + D + H = 7 or C * D * H = 7, we can determine that C must be 3. So the squares must look this:

8 A	? 2	7 3	D
E	4 F	8 G	H
I	J	K	8 4
7 3	4	1	2

With D and H being 1 and 3, it must be that D = 1 and H = 3, as we already have a 1 in the first row. Similarly, the third column is missing a 2 and a 4, but the 3rd row already has a 4, so G = 4 and K = 2.

^{8}A	$^{?}$2	^{7}3	1
E	^{4}F	^{8}4	3
I	J	2	^{8}4
^{7}3	4	1	2

From here, the first row needs a 4, so A = 4. The second row needs a 1 and a 2, so E = 2 and F = 1, as the second column already has a 2. And the third row needs a 1 and a 3, and they must be in that order as the first column already has a 3. So the squares must be:

^{8}4	$^{?}$2	^{7}3	1
2	^{4}1	^{8}4	3
1	3	2	^{8}4
^{7}3	4	1	2