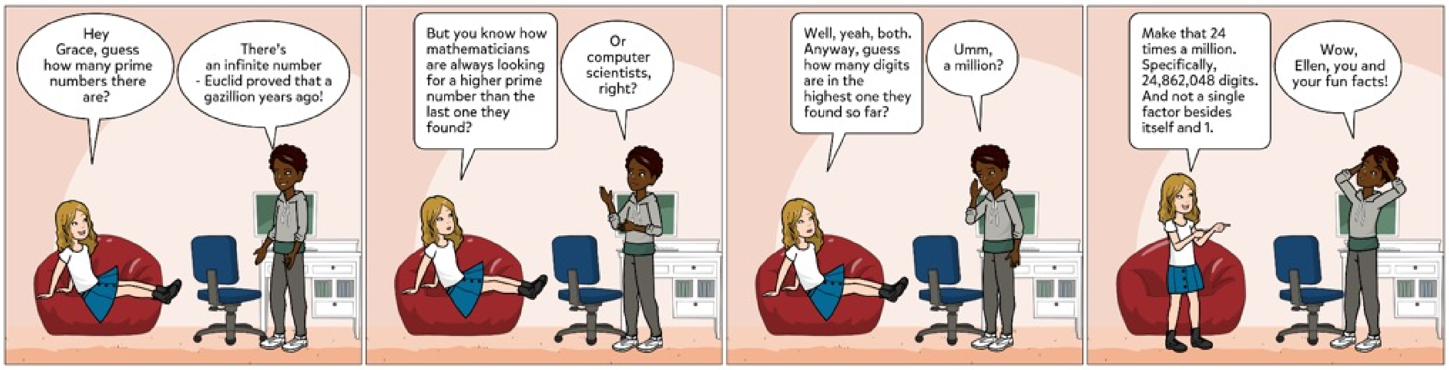
GAIM2023 NEWSLETTER #1

Prime Numbers & Prime Factorization

WITH ELLEN AND GRACE



What is a **prime** number?

2 97 89 43 29 5 101 7 67 13 79 37

A **prime number** is a positive integer that has exactly two positive factors: 1 and itself. It is not divisible by any other positive integer other than 1 and itself.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, ..., 89, 97, 101, ..., and so on.

More fun facts!

1. 0 and 1 are not prime numbers!
2. 2 is the smallest prime number, and the only even one.

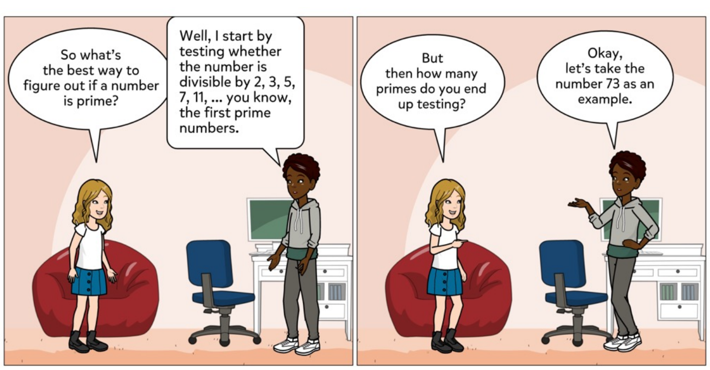
What is a **composite** number?

9 64 51 26 30 4 95 6 75 39 27 99

A **composite number** is a positive integer that is divisible by more than two positive integers.

Examples: 4 is the first composite number. It's divisible by 1, 2, and 4. The next ones are 6, 8, 9, 10, 12, ..., 98, 99, 100, ... and so on.

TRY IT YOURSELF!



Is 73 a prime number?

- It's not divisible by 2, so it's not divisible by 4, 6, 8, 10, or any other even number.
- It's not divisible by 3, so it's not divisible by 9 either.
- It's not divisible by 5 or 7.

Go to the next page for a hint! ▸

Here's a hint! Do we need to try dividing by any other number?

No, because by these tests, we have already covered all numbers that have factors of 2-10. The largest such number is 10*10=100. 73 is not any composite number between 2 and 100. Therefore it must be prime. Actually, we did not even need to test 10, because 9x9=81, and 73 is less than 81.

PRIME FACTORIZATION

Did you know that every composite integer can be broken down into a product of primes? For example, 60 = 2 x 2 x 3 x 5. When you split up a number into its prime factors, you are taking its **prime factorization**.

Examples:

$20 = 2 \times 2 \times 5 = 2^2 \times 5$

$42 = 2 \times 3 \times 7$

$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

According to the Fundamental Theorem of Arithmetic, any integer greater than 1 is either a prime number, or can be written as a unique product of prime numbers.

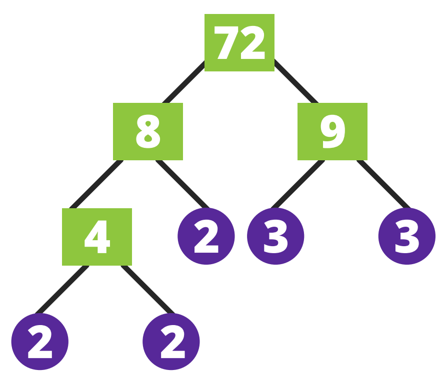
Building a Factor Tree

How do you find the prime factorization of an integer n > 1? One of the clearest and most foolproof ways is to build a factor tree:

1. If the number is prime, you are done!
2. Separate the number into a product of two smaller factors.
3. If one or more of those numbers is prime, circle it.
4. If the number(s) on the end of the branch is composite, continue by separating it into a product of two smaller factors.
5. Go back to step 3 and keep going until all the numbers on the ends of the branches are prime.

EXAMPLE

Start with 72:



Step 1: 72 is not prime.

Step 2: 72 = 9 x 8

Step 3: 9 and 8 aren't prime.

Step 4: 9 = 3 x 3, 8 = 2 x 4

Step 5, back to 3: The 3's are prime, and so is the 2. Circle them. 4 is composite.

Step 4 again: 4 = 2 x 2

Step 5 -> back to 3: The 2's are prime.

We have no other composite numbers at the ends of branches, just beautiful circled primes. We're done!

Write in a product. The prime factorization of 72 is $2^3 \times 3^2$.

Prime Palooza Practice Problems!
WORD PROBLEMS

Complete the following word problems.

1. Let's start with the basics. Find the prime factorizations of the following numbers. If the number is prime, write "prime".

a. 96

b. 103

c. 357

d. 625

e. 1001

2. Alicia's friend says, "I think 246,813,579 is prime." Is he right or wrong?

3. What is the smallest prime factor of 71,025?

4. What is the smallest composite number that is not divisible by 2, 3, 5, or 7?

5. What is the smallest odd number that has three different prime factors?

6. Write the prime factorization of 2 x 4 x 6 x 8 x 10 x 12. This should be in the form: x^a x y^b x z^c, where x, y, and z are different prime numbers and a, b, and c are exponents (powers) of those prime numbers.

7. A positive number has 3 digits. The product of the digits is 135. What is the sum of the digits?

8. Chien-Shiung is thinking of a 3-digit number. She multiplies the digits and gets 20. How many distinct 3-digit numbers could Chien-Shiung be thinking about?