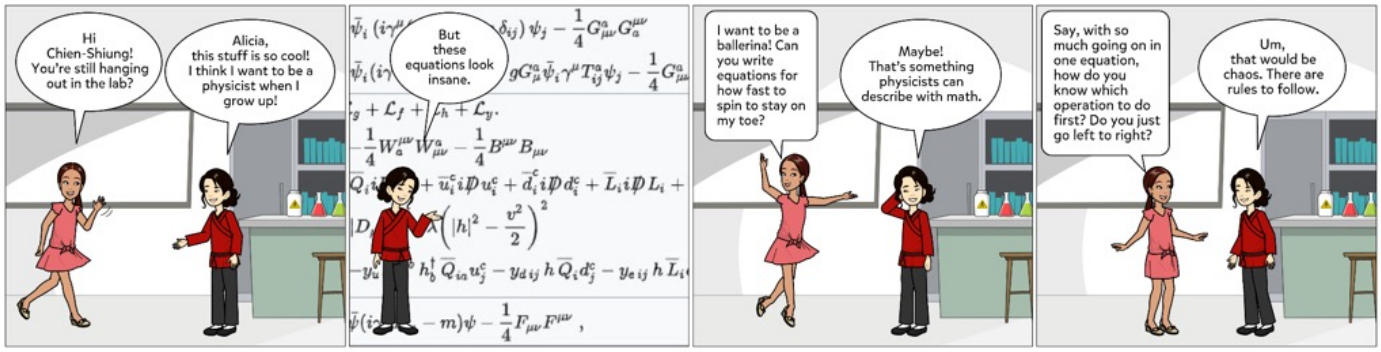
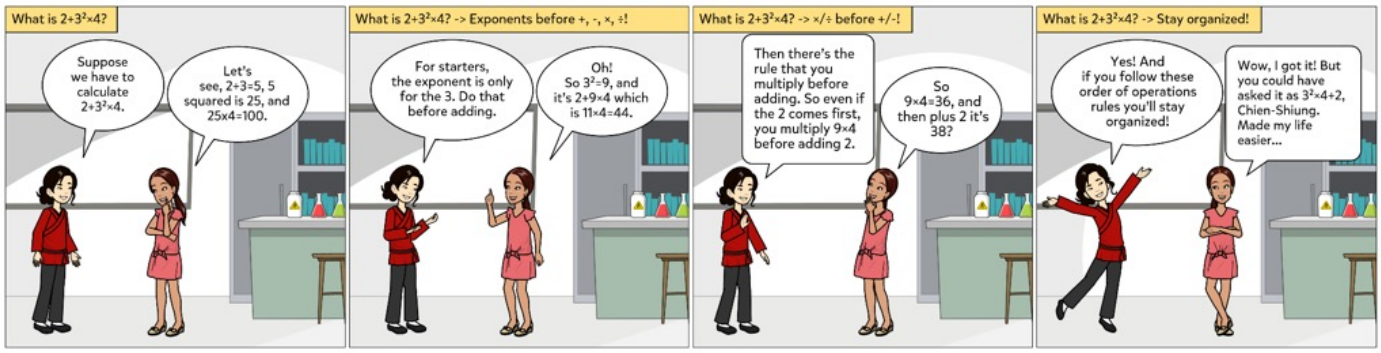
GAIM2023 NEWSLETTER #2

Operation Cooperation

WITH CHIEN-SHIUNG AND ALICIA



Getting to the answer is clear when an expression has only one operation, like **12** - **5.** But it can be confusing in an expression with multiple operations. So is there a proper order to do the operations in the expression **2** + **3²** x **4**?



Order of Operations Rules

In an expression with more than one operation, do them in this order:

1. Compute expressions inside parentheses first.
2. Compute exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

When were these rules established?

In the 1500's AD, the earliest algebra books used the convention that multiplication should be done before addition. Within the next century mathematicians started using exponents, which they decided should go before multiplication.

TRY IT YOURSELF!

How do we compute 180 ÷ 6² + (10 - 5)?

1. First evaluate the part in parentheses: 10 - 5 = 5.
2. There is an exponent. $6^2 = 36$. So far we have 180 ÷ 36 + 5.
3. Do the division next. 180 ÷ 36 = 5.
4. Do the addition last. 5 + 5 = 10. The final answer is 10!

The final answer is 10!

Invented Operators

Sometimes math problem writers like to invent operators and define them using the parentheses, exponents, operators. Suppose you get a question like this:

If **a ♥ b** is defined as **a x b + 2**, what is the difference between (**2 ♥ 4**) **♥ 5** and **2 ♥** (**4 ♥ 5**)?

- Take the number on the left side of the ♥ and plug it in for "**a**". Plug in the number on the right side of the ♥ for "**b**". (For example, **3 ♥ 5** would equal **3 x 5 + 2** = 17).
- Parentheses come first!
- The first expression comes out to (**2 x 4 + 2**) **♥ 5** = **10 ♥ 5** = **10 x 5 + 2** = **52**.
- The second expression comes to **2 ♥** (**4 x 5 + 2**) = **2 ♥ 22** = **2 x 22 + 2** = **46**.
- The difference is **52** - **46** = **6**.

TRY IT YOURSELF!

Can you come up with your own operation? Numbers are fun to play with, so get creative and see if your friends can evaluate expressions using your opeations!

Operation Cooperation Practice Problems!

WORD PROBLEMS

These practice problems can be solved either in teams (ideally) or individually. Exercise your operation skills, and when you are done, submit the Google form to be entered in a drawing for an Art of Problem Solving gift card! Entries received before February 14, 2023 will be eligible for the prize. You do not have to have registered yet for GAIM 2023 to enter the Operation Cooperation drawing.

1. Evaluate, using order of operations:

a. $17 - 5 \times (3 - 1)$

b. $10 + 101 \times 0 - 10 \div 2$

c. $4^2 - 7 \times 2$

d. $20 \div 2 \times (4 + 5)$

e. $100 - (2 \times (15 - 3))$

2. Which of the following values is largest?

a. $2 + 0 + 2 + 3$

b. $2 \times 0 + 2 + 3$

c. $2 + 0 \times 2 + 3$

d. $2 + 0 + 2 \times 3$

e. $2 \times 0 \times 2 \times 3$

3. The incorrect statement **$-5 + 2 \times 6 - (-2) = 15$** can be corrected by adding 1 to one of the numbers. Which number is it?

4. Add parentheses to the equation to make a true statement. Use order of operations!

a. $9 - 5 \times 2 - 4 = 3$

b. $9 - 5 \times 2 - 4 = 4$

c. $2 \times 9 - 6 + 4 = 8$

d. $2 \times 9 - 6 + 4 = 10$

e. $6 - 9 - 4 \times 2 = 5$

5. Define the operation 😍 as:

x 😍 $y = (x + y)^2 - 1$

a. If $x = 2$, what values of y will give x 😍 $y = 24$?

b. What is 5 😍 (-3) 😍 4?

6. In Addiatorsville, addition/subtraction go before multiplication/division in order of operations. What is the positive difference between the values in Addiatorsville and in our world of these expressions?

a. $4 \times 5 - 3 \times 3$

b. $(6 - 3 \times 5)^2$

c. $8 \div 2 + (51 - 7^2) \times 3$

7. If the operation ♦ is defined as:

s ♦ $t = s \times t - 2$

What is the value of (3 ♦ 4) ♦ (5 ♦ 6) ?

8. Alicia was asked by her teacher to subtract 4 from a certain number and then divide the result by 8. Instead, she subtracted 8 and then divided the result by 4, giving an answer of 23. What would her answer have been had she worked the problem correctly?