Solutions to Newsletter 1 Prime Palooza Practice Problems

1. **Let's start with the basics. Find the prime factorizations of the following numbers. If the number is prime, write "prime".**

   Solutions:

   a. $96 = 48 \cdot 2 = 24 \cdot 2 \cdot 2 = 12 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 \cdot 3$
   b. 103 = prime
   c. $357 = 3 \cdot 119 = 3 \cdot 7 \cdot 17$
   d. $625 = 5 \cdot 125 = 5 \cdot 5 \cdot 25 = 5 \cdot 5 \cdot 5 \cdot 5 = 5^4$
   e. $1001 = 7 \cdot 11 \cdot 13$

2. **Alicia's friend says, "I think 246,813,579 is prime." Is he right or wrong?**

   Solution: Try to find a divisor of 246,813,579. Obviously this number is odd but a quick check reveals that it is a multiple of 3 (2+4+6+8+1+3+5+7+9=45, which is a multiple of 3). Thus, this number is a multiple of 3 and, therefore, is composite. So Alicia's friend is wrong.

3. **What is the smallest prime factor of 71,025?**

   Solution: Test 71,025 for divisibility by small prime numbers. It's not divisible by 2, but it is divisible by 3 (7+1+0+2+5=15, which is a multiple of 3). Hence the smallest prime factor of 71,025 is 3.

4. **What is the smallest composite number that is not divisible by 2, 3, 5, or 7?**

   Solution: Consider the prime factorization of this number. Since it is not divisible by 2, 3, 5 or 7, all the primes in the prime factorization are 11 or greater. Since it is composite, the prime factorization consists of at least two primes. Hence, our number is $11 \cdot 11 = 121$.

5. **What is the smallest odd number that has three different prime factors?**

   Solution: Fun fact: An odd number can only have odd factors. The three smallest odd prime factors are 3, 5, and 7. Hence, the smallest odd number that has three different prime factors is $3 \cdot 5 \cdot 7 = 105$.

6. **Write the prime factorization of $2 \times 4 \times 6 \times 8 \times 10 \times 12$. This should be in the form:**

   $x^a \times y^b \times z^c$, where x, y, and z are different prime numbers and a, b, and c are exponents (powers) of those prime numbers.

   Solution: The primes that this number is divisible by are 2, 3, and 5.
   -- there is only one power of 5, the one that comes from 10.
   -- there are two powers of 3, one that comes from 6 and one that comes from 12.
   -- there is one power of 2 that comes from 2, 6 and 10. Two powers of 2 come from each of 4, and 12. And three powers of 2 come from 8.
   Therefore, there are a total of 3 + 4 + 3 = 10 powers of 2.
   Hence, the prime factorization is $2^{10} \times 3^2 \times 5^1$.

7. **A positive number has 3 digits. The product of the digits is 135. What is the sum of the digits?**

   Solution: $135 = 27 \cdot 5 = 3^3 \cdot 5 = 3 \cdot 9 \cdot 5$. So $3 + 9 + 5 = 17$ is the sum of the digits.
   Is this the only possible answer? Yes, because any other combination requires one of the other 3 factors to be a 2-digit number, which can't be used as a digit.

8. **Chien-Shiung is thinking of a 3-digit number. She multiplies the digits and gets 20. How many distinct 3-digit numbers could Chien-Shiung be thinking about?**

   Solution: Note that 20 = 5 * 4. Hence one of the digits in Chien-Shuing's number is a multiple of 5. If that digit were zero, then the product would also be zero and not 20.
   Therefore, 5 is one of Chien-Shiung's digits.
   The other two digits multiply to 4. They are either 1 and 4 or 2 and 2.
   If they are 1 and 4, there are 6 ways to rearrange 1,4 and 5.
   If they are 2 and 2, there are only 3 ways to arrange 2,2, and 5.

   Hence, there are a total of 6 + 3 = 9 possibilities for Chien-Shiung's number.